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A NOVEL METHOD OF HANDLING TOLERANCES FOR ANALOG CIRCUIT FAULT DIAGNOSIS BASED ON NORMAL QUOTIENT DISTRIBUTION

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Abstract

While the Slope Fault Model method can solve the soft-fault diagnosis problem in linear analog circuit effectively, the challenging tolerance problem is still unsolved. In this paper, a proposed Normal Quotient Distribution approach was combined with the Slope Fault Model to handle the tolerances problem in soft-fault diagnosis for analog circuit. Firstly, the principle of the Slope Fault Model is presented, and the huge computation of traditional Slope Fault Characteristic set was reduced greatly by the elimination of superfluous features. Several typical tolerance handling methods on the ground of the Slope Fault Model were compared. Then, the approximating distribution function of the Slope Fault Characteristic was deduced and sufficient conditions were given to improve the approximation accuracy. The monotonous and continuous mapping between Normal Quotient Distribution and standard normal distribution was proved. Thus the estimation formulas about the ranges of the Slope Fault Characteristic were deduced. After that, a new test-nodes selection algorithm based on the reduced Slope Fault Characteristic ranges set was designed. Finally, two numerical experiments were done to illustrate the proposed approach and demonstrate its effectiveness.

Keywords: soft-fault diagnosis, analog circuit, Normal Quotient Distribution, Slope Fault Model.

1. Introduction

Fault detection, fault location and fault identification are three major tasks of analog circuit fault diagnosis [1]. According to collected statistics, although most parts of an electronic system are digital, more than 80% of the faults occur in the analog segment. The analog circuits fault diagnosis can be an extremely difficult problem due to 1) the lack of good fault models for analog components similar to the stuck-at-one and stuck-at-zero fault models, which are widely used by the digital circuit test; 2) the nonlinear nature of the problem; 3) component tolerances. For these reasons, the test and diagnosis of analog circuits are still developing very slowly.

On the basis of simulations of the Circuit Under Test (CUT) occurring in a test, fault diagnosis techniques are classified as the Simulation Before Test approach (SBT) and the Simulation After Test approach (SAT). In the SAT approach, fault diagnosis is achieved by calculating the circuit parameters directly from the measured responses of CUT. It needs more computation in test than the SBT approach. The fault dictionary method is a typical case of SBT, in which the simulations need to be implemented before the test to generate an off-line fault dictionary for the predefined fault states. Given the strong requirements of shortest test times and the best test coverage in today's semiconductors test [2], the fault dictionary is becoming one of the most promising fault diagnosis methods.

The fault dictionary technique can only be used to diagnose the hard-fault (also being called catastrophic-fault, including open fault and short fault) in the early time [1, 3]. Recently, it has been improved to diagnose both the hard-fault and soft-fault (also being

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called parameter-fault, when the faulty element deviates from its nominal value beyond its tolerance limit). In [4-6], node-voltage sensitivity analysis methods were used to diagnose a soft-fault. A novel fault dictionary based on the conservation of node voltage sensitivity weight sequence was constructed in [6]. The linear relationships between variations in node voltages were discovered by Wang and Yang in [7]. The Slope Fault Model (SFM) was proposed by Wang and Yang in reference [8]. Whether a fault is hard or soft, only one Slope Fault Characteristic (SFC) is needed. However, the diagnosis accuracy under tolerances was low for adopting the Minimal Distance Approach (MDA). In [9], Yang and Tian improved the method in [8] to handle tolerances by the Fixed Width Approach (FWA). Acting as a feature selector and classifier, the Self-organizing Map was used to diagnose a soft-fault under tolerances in analog electronic circuits [10]. The fuzzy math concept was introduced to handle tolerances by [11-13]. In [13], the direction vector fault signature was combined with fuzzy analysis method to deal with tolerances in CUT. In recent years, the Support Vector Machine was introduced as pattern classifier for diagnosing analog circuit [14]. Tadeusiewicz and Hałagas had developed interesting algorithms based on the linear programming method, which can diagnose multiple soft-faults without performing any optimization process [15-16]. Most of the aforementioned fault diagnosis approaches based on a fault dictionary can partly solve the tolerances problem in fault diagnosis of an analog circuit, but the diagnosis coverage is low.

In this paper, a novel approach based on Normal Quotient Distribution (NQD) was proposed to deal with the tolerance problem in analog circuit fault diagnosis. The distribution function of the SFC was established. An approximating calculation method of the above distribution function was developed. The estimating equations of the SFC with high accuracy were given. An effective test-node selection algorithm was designed. This paper is organized as follows below.

Section 2 illustrates the principle of the SFM and proves an effective SFC selection method. Section 3 compares several typical approaches of handling tolerances based on SFM in fault diagnosis, and explains the NQD approach. Section 4 shows the fault diagnosis process based on a new test-node selection algorithm. Section 5 presents two experiments to demonstrate the effectiveness of the proposed method. Section 6 draws the conclusions.

2. The principle of the SFM

Figure 1a shows a linear CUT network stimulated by an independent voltage source u_s with n components. Let us suppose that the test-nodes set and the faults set of CUT are $T=\{T_1,T_2,...,T_p\}$ and $F=\{F_0,F_1,F_2,...,F_m\}(1\le m\le n)$, where F_0 means the fault-free state and $F_j(j=1,2,...,m)$ means X_j is the possible faulty component. N is the number of non-faulty parts of the CUT with component parameters being their nominal values.

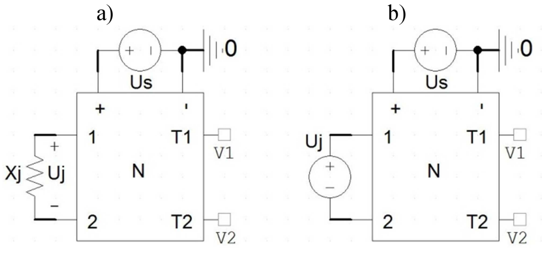

Fig. 1. a) CUT network; b) equivalent network.

Subject to the substitution theorem [17], Fig. 1a can be equivalently transformed into Fig. 1b if the voltage decrease of X_j is u_j. In Fig. 1b, CUT is stimulated by both u_s and u_j.

Based on the superposition theorem of a linear circuit [17], the test-node voltage is the algebraic sum of two parts: the first is V^{us} caused by u_s while u_j is short-circuited, the other is V^{uj} caused by u_j while u_s is short-circuited.

The test-nodes voltages caused only by u_s are:

$$V_1^{us} = a_{1s}u_s\,,\;\; V_2^{us} = a_{2s}u_s\,, \tag{1}$$

where a_{1s} and a_{2s} are transmission coefficients between u_s and T_1, T_2. The test-nodes voltages caused only by u_j are:

$$V_1^{uj} = a_{1j}u_j\,,\;\; V_2^{uj} = a_{2j}u_j\,, \tag{2}$$

where a_{1j} and a_{2j} are transmission coefficients between u_j and T_1, T_2. The total voltages are:

$$V_1^j = a_{1s}u_s + a_{1j}u_j\,,\;\; V_2^j = a_{2s}u_s + a_{2j}u_j\,, \tag{3}$$

Eliminate u_j in (3):

$$V_1^j = \frac{a_{1j}}{a_{2j}}V_2^j + a_{1s}u_s - \frac{a_{1j}}{a_{2j}}a_{2s}u_s = S_{12}^j V_2^j + u_c\,, \tag{4}$$

where $a_{1j}, a_{2j}, a_{1s}, a_{2s}$ are parameters only affected by the locations of the test-nodes, the location of X_j, components parameters and topology in N. If u_s is constant, whatever X_j is (open, short, soft-fault), $S_{12}^j = a_{1j}/a_{2j}$ and $u_c = a_{1s}u_s - a_{1j}a_{2s}u_s/a_{2j}$ are constant. It is reasonable to choose S_{12}^j to be the SFC of F_j.

When the parameter of X_j is at the nominal value, the voltages at T_1 and T_2 should meet (4), which can be shown in (5):

$$V_1^0 = S_{12}^j V_2^0 + u_c\,. \tag{5}$$

Subtracting (5) from (4), (6) can be acquired.

$$(V_1^j - V_1^0) = S_{12}^j (V_2^j - V_2^0) \Rightarrow \frac{\Delta V_1^j}{\Delta V_2^j} = S_{12}^j\,. \tag{6}$$

S_{12}^j can be computed numerically from (6).

Take the CUT shown in Fig. 2 for example. When R_1 is the faulty component (open fault ($10^6\,\Omega$), short fault (0.001 Ω) and soft-fault (8 kΩ)), the voltage variations at T_1 and T_2 are (−8.01 V,7.98 V,−2.67 V) and (−4.03 V,4.02 V,−1.34 V) respectively. S_{12}^1 are calculated to be (0.5029,0.5030,0.5028). Whatever R_1 is, open, short or soft-fault, S_{12}^1 keep to be the same.

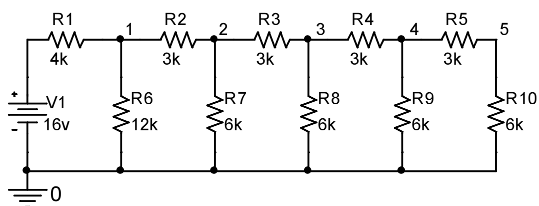

Fig. 2. Voltage divider circuit.

To diagnose a CUT with m possible fault states and p test-nodes, there totally are $mp(p-1)$ SFC needed to be computed before test [9]. The huge computation is unacceptable in practice. For most of the SFC carrying redundant information, there is an effective method to reduce the computation.

Theorem 1

For a CUT with p test-nodes, there are only $(p-1)$ effective SFC while CUT is at any fault state.

Prove: Suppose that T_k is the basic test-node, which can be determined in Section 4.1, we can construct $(p-1)$ SFC $S_{ik}(i=1,2,...,p,i\neq k)$. As it is shown in (7), all the other $(p-1)^2$ SFC $S_{ij}(i\neq j, j\neq k)$ can be got directly using S_{ik}.

$$\frac{S_{ik}}{S_{jk}}=(\frac{\Delta V_i}{\Delta V_k})/(\frac{\Delta V_j}{\Delta V_k})=\frac{\Delta V_i}{\Delta V_j}=S_{ij}\ ,\ (i,j=1,2,...,p,i\neq j\neq k). \tag{7}$$

Clearly, S_{ij} should be eliminated from the effective SFC set for the linear correlation to S_{ik}/S_{jk}. Consequently, the previously described computation becomes $m(p-1)$.

3. Methods of handling tolerances based on SFM

Tolerance problem is one of the most challenging problems in analog circuit fault diagnosis. Let $\overline{S_{ij}^k}$ denotes the nominal SFC between T_i and T_j under F_k while all the parameters of the non-faulty components are right at their nominal values. Clearly, $\overline{S_{ij}^k}$ are constant. If tolerances are taken into account, which are inevitable in analog circuits, S_{ij}^k are Random Variables (RVs) with their parameters changing around $\overline{S_{ij}^k}$.

3.1. Comparison of typical tolerance handling approaches

There are two sorts of approaches to handle tolerances in fault diagnosis for analog circuits. One sort is the non-probabilistic method, including the MDA [8] and FWA [9, 13]. The other sort is the probabilistic method, including the 3σ approach. The fuzzy analysis methods will not be discussed in this paper.

Because only nominal SFC need to be calculated, MDA is an algorithm with least computation. At the same time, it is a rough algorithm for fault diagnosis under tolerances. For example, let $\overline{S^1}=1$ and $\overline{S^2}=0$ be the nominal SFC for two fault states F_1 and F_2, and the ranges of the corresponding SFC are (0.1,1.9) and (−0.1,0.1) respectively. If the measured SFC is $S=0.2$, the diagnosis result will be F_2 by MDA. In fact, F_1 is the right fault state.

The FWA is more accurate than MDA by taking the SFC range caused by tolerances into account. By this way, the SFC range is expressed as $S_{FWA}=(S_{nom}(1-\beta),S_{nom}(1+\beta))$, where S_{nom} is the nominal SFC and β is a specified width factor. While the simulation computation of FWA is small, it is not logical to impose a strong correlation between S_{nom} and S_{FWA}. For example, if $S_{nom}\to 0$, then $S_{FWA}\to 0$, which means there are no tolerances in CUT at all.

Given normal distribution is the most common probability distribution in the real world, the SFC are considered directly to be RVs with normal distribution in the 3σ approach (actually it is NQD). On this hypothesis, the SFC range is $S_{3\sigma}=(S_{nom}-3\sigma,S_{nom}+3\sigma)$.

Though $S_{3\sigma}$ is wide enough to cover nearly all the SFC under tolerances, it is not helpful to the optimum test-node selection.

According to all the disadvantages mentioned above, a novel method of handling tolerances based on NQD is put forward.

3.2. NQD approach

Suppose all the parameters of the non-faulty components in CUT are independent normal RVs with mean values at their nominal values. Subject to the Law of Central limit theorem [i8], the voltages variation at the test-nodes are also normal RVs. In view of the SFC definition in (6), the distribution of SFC between two test-nodes is NQD.

For the convenience of computing, X and Y, with means as θ_x and θ_y, variances as σ_x^2 and σ_y^2, and correlation coefficient ρ, are used to represent the voltage variation on two test-nodes in CUT. W denotes the corresponding SFC.

Hypothesis 1: $\theta_x \neq 0, \theta_y \neq 0, \sigma_x \neq 0, \sigma_y \neq 0$.

Under the above assumptions, the relation between the three random variables is shown in (8):

$$W = \frac{X}{Y}. \tag{8}$$

If the joint density function of (X,Y) is $g(x,y)$ and the density function of W is $f(w)$, then:

$$f(w) = \int_{-\infty}^{\infty} |y|\, g(wy, y)dy. \tag{9}$$

The distribution function $F(w)$ of W is found by direct calculation to be [19-20]:

$$F(w) = L\{\frac{\theta_x - \theta_y w}{\sigma_x\sigma_y a(w)}, -\frac{\theta_y}{\sigma_y}, \frac{\sigma_y w - \rho\sigma_x}{\sigma_x\sigma_y a(w)}\} + L\{\frac{\theta_y w - \theta_x}{\sigma_x\sigma_y a(w)}, \frac{\theta_y}{\sigma_y}, \frac{\sigma_y w - \rho\sigma_x}{\sigma_x\sigma_y a(w)}\}, \tag{10}$$

where $L\{h,k,r\} = P\{\xi > h, \eta > k\}$, ξ and η are RVs of standard normal distribution with covariance r.

Equation (10) is easy to be simplified to (11) and (12):

$$F_1(w) = \Phi(\frac{\theta_y w - \theta_x}{\sigma_x\sigma_y a(w)}),\ \frac{\theta_y}{\sigma_y} \to \infty, \tag{11}$$

$$F_2(w) = \Phi(\frac{\theta_x - \theta_y w}{\sigma_x\sigma_y a(w)}),\ \frac{\theta_y}{\sigma_y} \to -\infty, \tag{12}$$

where $a(w) = (\frac{w^2}{\sigma_x{}^2} - \frac{2\rho w}{\sigma_x\sigma_y} + \frac{1}{\sigma_y{}^2})^{\frac{1}{2}}$, $\Phi(\bullet)$ is the distribution function of standard normal distribution. On account of the 3σ ($\sigma = 1$, σ^2 is variance of standard normal distribution) law of standard normal distribution, the two approximations are accurate enough if $|\theta_y / \sigma_y| > 3\sigma = 3$. According to the high similarity of (11) and (12), only (11) is used to calculate the range of W. Let:

$$z = h(w; \theta_x, \theta_y, \sigma_x, \sigma_y, \rho) = \frac{\theta_y w - \theta_x}{\sigma_x\sigma_y a(w)}. \tag{13}$$

Clearly, Z is a RV of standard normal distribution. It is easy to prove that the numerator and the denominator of equation (13) are continuous functions and the denominator is positive. Consequently, $h(\bullet)$ is a continuous mapping.

Theorem 2

Within a certain range, there is monotonous and continuous mapping between z and w.

Prove: Analysis shows that there is quadratic function mapping between z and w in the whole domain of w. Let us consider firstly the extreme points of z which can be calculated by (14):

$$\frac{dz}{dw} = 0 \Rightarrow w_{dz=0} = -\frac{\theta_y\sigma_x^2 - \rho\theta_x\sigma_x\sigma_y}{\sigma_y(-\rho\theta_y\sigma_x + \theta_x\sigma_y)}. \tag{14}$$

Clearly, there is only one extreme point $(w_{dz=0})$ for specified $(\theta_x, \theta_y, \sigma_x, \sigma_y, \rho)$. $\frac{dz}{dw}$ keeps to be positive or negative on each side of $w_{dz=0}$. That is to say, z changes monotonously and continuously with w on each side of $w_{dz=0}$. But the monotonous mapping is not satisfied in the neighbourhood of $w_{dz=0}$. Substituting (14) back into (13), the extreme value of z can be computed from (15):

$$\left|z_{dz=0}\right| = \left|\frac{(\rho\theta_y\sigma_x - \theta_x\sigma_y)\sqrt{\dfrac{(1-\rho^2)(\theta_y^2\sigma_x^2 - 2\rho\theta_x\theta_y\sigma_x\sigma_y + \theta_x^2\sigma_y^2)}{\sigma_y^2(\rho\theta_y\sigma_x - \theta_x\sigma_y)^2}}}{(1-\rho^2)\sigma_x}\right|$$

$$= \left|\frac{\sqrt{\theta_y^2\sigma_x^2 - 2\rho\theta_x\theta_y\sigma_x\sigma_y + \theta_x^2\sigma_y^2}}{\sqrt{1-\rho^2}\,\sigma_x\sigma_y}\right| > \left|\frac{\theta_y\sigma_x - \rho\theta_x\sigma_y}{\sqrt{(1-\rho^2)}\sigma_x\sigma_y}\right| > \left|\frac{\theta_y\sigma_x - \rho\theta_x\sigma_y}{\sigma_x\sigma_y}\right| = \left|\frac{\theta_y}{\sigma_y} - \frac{\rho\theta_x}{\sigma_x}\right| \tag{15}$$

On the condition that $\left|\theta_y / \sigma_y\right| \to \infty$, $z_{dz=0}$ must be a large number away from its mean $z = 0$. Due to the nature of standard normal distribution, the probability of z belonging to the neighbourhood of $z_{dz=0}$ is very small. Accordingly, the probability of w belonging to the neighbourhood of $w_{dz=0}$ is also very small.

Then, let us consider the zero points of z which can be calculated by (16)

$$z = 0 \Rightarrow w_0 = \frac{\theta_x}{\theta_y}. \tag{16}$$

$z = 0$ is the mean value of Z. According to probability theory, the corresponding w_0 is the mean value of W. There is only one zero point (w_0) for specified $(\theta_x, \theta_y, \sigma_x, \sigma_y, \rho)$.

Above all, W changes on one side of $w_{dz=0}$, which includes w_0 (Only this range is needed to be considered in this paper), and is seldom equal to $w_{dz=0}$. There is monotonous and continuous mapping between z and w in this range. The variation range of W can be estimated below.

Suppose $\alpha\%$ of $z \in (-z_1, z_1), z_1 > 0$, then z_1 can be got from (17):

$$P(-z_1 < Z < z_1) = \Phi(z_1) - \Phi(-z_1) = \alpha\%. \tag{17}$$

If $\alpha = 90$, then $z_1 = 1.645$. Putting z_1 and $-z_1$ back into (13), the corresponding range (w_1, w_2) can be calculated easily from (18):

$$w_{1,2} = \frac{2\theta_x\theta_y - 2z_1^2\rho\sigma_x\sigma_y \pm \sqrt{(-2\theta_x\theta_y + 2z_1^2\rho\sigma_x\sigma_y)^2 - 4(\theta_x^2 - z_1^2\sigma_x^2)(\theta_y^2 - z_1^2\sigma_y^2)}}{2(\theta_y^2 - z_1^2\sigma_y^2)}. \tag{18}$$

Let us take the two groups of computer-generated data $X = N(1, 0.5^2)$ and $Y = N(3, 0.8^2)$ for example. The ranges calculated by FWA, 3σ approach and (18) are $S_{FWA} = (0.3, 0.3667)(\beta = 0.1)$, $S_{3\sigma} = (-0.263, 0.9297)$ and $S_{NQD} = (0.0565, 0.6884)$ respectively. $S_{3\sigma}$ contains nearly 100% SFC under tolerances, but the range is too big. S_{FWA} is apparently the best, but it only contains less than 30% SFC. For better balance between small range and high coverage of SFC, S_{NQD} (contains more than 95% SFC) is chosen to be the reasonable estimation of SFC range under tolerances.

4. Diagnosis methodology and process

Based on the SFM and NQD approach, we can handle open fault, short fault and soft-fault of CUT under tolerances. The general diagnosis steps are as follows

Step 1. Run nominal simulations (CUT is in F_j states while all the non-faulty components are at their nominal values) to get $V_i^j (i = 1, 2, ..., p; j = 1, 2, ..., m)$, then calculate ΔV_i^j;

Step 2. Run Monte Carlo (MC) simulations (CUT is in F_j states while all the non-faulty components change around the nominal values within their tolerance limits) to get V_{iq}^j (q is MC simulations number for each fault states);

Step 3. Calculate ΔV_{iq}^j and $(\theta_i^j, \sigma_i^j, \rho_{ik}^j)$, where θ_i^j and σ_i^j denote the means and variances of ΔV_{iq}^j, ρ_{ik}^j denotes the correlation coefficient between ΔV_{iq}^j and ΔV_{kq}^j, then determine the basic test-node T_k;

Step 4. Calculate the ranges of the SFC $(S_{ik}^{j1}, S_{ik}^{j2})$ by (18);

Step 5. Select the optimum test-nodes and construct a fault dictionary;

Step 6. Fault diagnosis.

MC simulations are the key operations for acquiring the statistics of the SFC under tolerances. On account of 1) rapid development of computer technique, 2) MC simulations are needed only once before test, 3) the more accurate the SFC ranges are estimated, the less computations are needed in fault diagnosis, it is necessary to run it to improve the estimation of SFC ranges.

4.1. Basic test-node selection

For better approximation of (10) by (11) and (12), the basic test-node T_k must be determined. There are two conditions needed to be satisfied. Firstly, T_k should meet inequality (19):

$$\min_j \left|\theta_k^j / \sigma_k^j\right| > 3, (j = 1, 2, .., m; k \in \{1, 2, ..., p\}). \tag{19}$$

This condition makes the approximation feasible. Generally, more than one test-node satisfied the above inequality. Let K denotes the basic test-nodes set generated in this step.

Secondly, T_k should satisfy (20):

$$k = \arg\max_{k\in K} \underset{j}{sum}\left|\theta_k^j / \sigma_k^j\right|, (j = 1,2,..,m). \quad (20)$$

This condition makes the approximation more accurate and results in the only T_k.

4.2. Optimum test-node selection

The effective SFC can be constructed as $S_{ik}(i \in \{1,2,...,p\}; i \neq k)$ on the ground of the selected T_k. For each fault state F_j, the corresponding SFC ranges can be computed by (18). The SFC ranges table is constructed as Table 1.

Table 1. SFC ranges table.

	S_{1k}	S_{2k}	...	$S_{(p-1)k}$
F_1	$(S_{1k}^{11}, S_{1k}^{12})$	$(S_{2k}^{11}, S_{2k}^{12})$	...	$(S_{(p-1)k}^{11}, S_{(p-1)k}^{12})$
F_2	$(S_{1k}^{21}, S_{1k}^{22})$	$(S_{2k}^{21}, S_{2k}^{22})$	...	$(S_{(p-1)k}^{21}, S_{(p-1)k}^{22})$
...	...	...	...	...
F_m	$(S_{1k}^{m1}, S_{1k}^{m2})$	$(S_{2k}^{m1}, S_{2k}^{m2})$	...	$(S_{(p-1)k}^{m1}, S_{(p-1)k}^{m2})$

$(S_{ik}^{j1}, S_{ik}^{j2})$ means a SFC range between T_i and T_k under fault F_j.

Based on Table 1, $(p-1)$ Fault-pair Isolation Matrixes (FIM) can be generated as $(FIM_{1k}, FIM_{2k}, ..., FIM_{(p-1)k})$. FIM_{ik} corresponding to S_{ik} is shown in Table 2.

Table 2. FIM_{ik} (for S_{ik}).

	F_1	F_2	...	F_m
F_1	0	0	...	0
F_2	FIM_{ik}^{21}	0	...	0
...	...	...	...	...
F_m	FIM_{ik}^{m1}	FIM_{ik}^{m2}	...	0

FIM can be acquired by (21):

$$FIM_{ik}^{jl} = \begin{cases} 1 & (S_{ik}^{j1}, S_{ik}^{j2}) \bigcap (S_{ik}^{l1}, S_{ik}^{l2}) = \varphi \\ 0 & others \end{cases} \quad (i \in \{1,2,...,p\}; i \neq k; j,l = 1,2,...,m; j > l), \quad (21)$$

where φ means an empty set. Clearly, the estimated ranges $(S_{ik}^{j1}, S_{ik}^{j2})$ and $(S_{ik}^{l1}, S_{ik}^{l2})$ are bigger, the probability of $FIM_{ik}^{jl} = 1$ is smaller, which means F_j and F_l can be isolated by S_{ik}. This is why the 3σ approach is not helpful to the optimum test-nodes selection. For a CUT with m fault states, there are totally $m(m-1)/2$ fault-pairs (A group with two different fault states of CUT. For example, (F_1, F_2) is a fault-pair) to be isolated. For the purpose of isolating more fault-pairs with less test-nodes, an efficient test-node selection algorithm is designed below:

Step 1. Construct FIM by (21).

Step 2. The optimum test-nodes index set T^{opt} is initialized as $\{k\}$.

Step 3. Find out FIM_{ik} ($i \in \overline{T^{opt}}$, $\overline{T^{opt}} \bigcup T^{opt} = T$) with maximum number of 1, which means that the maximum fault-pairs are isolated, and add i into T^{opt} . Then save FIM_{ik} as a temporary matrix M^{TEM} .

Step 4. If all the fault-pairs are isolated, exit. Else, go to step 5.

Step 5. Calculate and find out $M^{TEM} \bigcup FIM_{ik} (i \in \overline{T^{opt}})$ with maximum number of 1, and add i into T^{opt} . Let $M^{TEM} = M^{TEM} \bigcup FIM_{ik}$.

Step 6. If all the fault-pairs are isolated, exit. Else, if no more fault-pairs are isolated, find out the undistinguishable fault-pairs, exit. Else, go to step 5.

T^{opt} is the optimum test-node set on the basis of the estimated SFC ranges in Table 1.

5. Experiment results and discussion

In this section, two experiments are designed to demonstrate the effectiveness of the proposed diagnosis method. The simulations and the data analysis were implemented with PSPICE and MATLAB respectively on a PC platform with AMD Athlon X2@2.8GHz and 2.0G RAM.

5.1. The voltage divider circuit

Let us consider a linear dc circuit (Fig. 2) first. There are totally 5 test-nodes $\{T_1, T_2, ..., T_5\}$ and 10 possible fault states $\{F_1, F_2, ..., F_{10}\}$, where F_j means R_j is the possible faulty component. The parameters of the faulty components increase 100% of their nominal value. Tolerances of all the components are 10%.

From the MC simulations for the predefined faults on CUT under tolerances, $(\theta_i^j, \sigma_i^j, \rho_{ik}^j)$ can be acquired by statistical calculations. Then, the basic test-node is determined to be T_5. After that, the nominal simulations need to be done and the nominal SFC (Table 3) can be calculated by (6). On the ground of the calculated $(\theta_i^j, \sigma_i^j, \rho_{ik}^j)$, the SFC ranges $(S_{ik}^{j1}, S_{ik}^{j2})$ (Table 3) can be computed from (18).

Table 3. SFC ranges for voltage divider circuit.

	nominal SFC				ranges of SFC			
	$\overline{S_{15}}$	$\overline{S_{25}}$	$\overline{S_{35}}$	$\overline{S_{45}}$	S_{15}	S_{25}	S_{35}	S_{45}
F_1	10.69	5.37	2.75	1.5	9.79~11.78	4.91~5.99	2.59~2.94	1.39~1.57
F_2	-5.31	5.38	2.76	1.5	-7.09~-3.91	4.74~6.16	2.49~3.08	1.41~1.6
F_3	-1.31	-2.63	2.75	1.5	-2.32~-0.43	-3.67~-1.79	2.55~2.99	1.42~1.59
F_4	-0.31	-0.63	-1.25	1.5	-1.35~0.58	-1.37~-0.01	-1.83~-0.79	1.4~1.62
F_5	-0.06	-0.13	-0.25	-0.5	-1.36~1.17	-1.21~0.77	-0.85~0.21	-1~-0.15
F_6	10.69	5.39	2.74	1.49	6.64~21.15	3.47~9.99	1.94~4.33	1.14~2.03
F_7	2.69	5.38	2.75	1.5	1.55~3.99	4.3~7.09	2.33~3.36	1.33~1.74
F_8	0.69	1.37	2.75	1.5	-0.38~1.66	0.64~1.99	2.41~3.18	1.35~1.66
F_9	0.18	0.38	0.75	1.5	-0.97~1.21	-0.31~0.95	0.38~1.05	1.38~1.64
F_{10}	0.06	0.13	0.25	0.5	-0.69~0.68	-0.37~0.51	-0.09~0.49	0.32~0.62

There are no correlations between the nominal SFC and their corresponding ranges (Table 3). For example, the range of S_{15} is 19% of its nominal value under fault F_1, but about 627% of the nominal value under fault F_4. This is why FWA is not logical.

Test-node selection: On the basis of Table 3, the FIM are obtained from (21). The proposed test-node selection algorithm is used to search the optimum test-nodes set T^{opt}. There are totally $m(m-1)/2 = 45$ fault-pairs. T_3 was selected out firstly to be the most informative test-node, because 35 fault-pairs can be isolated by S_{35}. Being the second informative test-node, T_1 was selected out to isolate other 5 fault-pairs. Then T_4 was selected out to isolate another 4 fault-pairs. There is only (F_1, F_6) that cannot be isolated. In fact, the nominal SFC vectors under fault F_1 and F_6 (Table 3) are too close to be distinguished. The final optimum test-nodes set $T^{opt} = \{T_1, T_3, T_4, T_5\}$. The corresponding fault dictionary based on T^{opt} can be constructed as Table 4.

Table 4. Fault dictionary for voltage divider circuit.

	S_{15}	S_{35}	S_{45}
F_1	9.79~11.78	2.59~2.94	1.39~1.57
F_2	-7.09~-3.91	2.49~3.08	1.41~1.6
F_3	-2.32~-0.43	2.55~2.99	1.42~1.59
F_4	-1.35~0.58	-1.83~-0.79	1.4~1.62
F_5	-1.36~1.17	-0.85~0.21	-1~-0.15
F_6	6.64~21.15	1.94~4.33	1.14~2.03
F_7	1.55~3.99	2.33~3.36	1.33~1.74
F_8	-0.38~1.66	2.41~3.18	1.35~1.66
F_9	-0.97~1.21	0.38~1.05	1.38~1.64
F_{10}	-0.69~0.68	-0.09~0.49	0.32~0.62

Four out of five nodes are needed to be accessible for CUT. This is often unrealistic. Fortunately, the selected test-nodes in T^{opt} are very different. T_5 is the indispensable basic-node. (T_5, T_3) form the most informative SFC, which can isolate 35 fault-pairs. About 80% fault-pairs are isolated by S_{35}. Then (T_5, T_1) form the second informative SFC (isolate another more 5 fault-pairs). Nearly 90% of the fault-pairs can be isolated by the three test-nodes. T_4 could be eliminated from T^{opt} for the limitation of the accessible nodes. As a result, the diagnosis accuracy is sacrificed.

There is another way to reduce the number of the accessible test-nodes. In the proposed approach, compromise must be reached between the diagnosis coverage and the number of accessible nodes. In (17), if α is big (means high diagnosis coverage), then z_1 is big.

Equation (18) shows that big z_1 leads to big (w_1, w_2) (the ranges of the SFC), which makes the fault-pairs of CUT hard to be isolated. Table 5 illustrates the relations between α and the accessible nodes. The data in Table 5 are the number of the separable fault-pairs. $\alpha = 0.9$ is adopted in this paper. If we make $\alpha = 0.7$, only (T_5, T_3) is needed to be accessible nodes, which can isolated nearly 90% of the fault-pairs. Consequently, the diagnosis coverage will drop.

Table 5. The relations between α and the accessible nodes.

	(T_5,T_3)	(T_5,T_1)	(T_5,T_4)	total
$\alpha=0.9$	35	5	4	44
$\alpha=0.8$	37	5	2	44
$\alpha=0.7$	40	4	0	44

Fault diagnosis: Suppose R_2 is the faulty component, which means CUT is under F_2 fault state. There are 6 fault types (4 soft faults, 1 open fault and 1 short fault) with the parameters being $\{1.3R_2, 1.5R_2, 2R_2, 5R_2, 10^6, 0.001\}$ Ω respectively. The parameters of the non-faulty components change around their nominal values independently within the tolerance limits (10%). The output voltages of $T_i (i=1,2,...,5)$ and the SFC calculated by (6) under F_2 are both listed in Table 6.

While the parameters of the faulty components are $\{1.3R_2, 1.5R_2, 2R_2, 5R_2, 10^6, 0.001\}$ Ω respectively, the corresponding SFC under tolerances (Table 6) are all in the SFC ranges of F_2 fault state (Table 4). Clearly, the diagnosis results are all F_2 fault state. When the parameter of the faulty component changes slightly ($1.3R_2$ Ω), one of the corresponding SFC (S_{35}) is not in the SFC ranges of F_2 fault state. The faulty component cannot be located correctly. For the hard-fault and the soft-fault with deviations greater than or equal to 50% from nominal values, the proposed method can locate the fault component accurately.

Table 6. Diagnosis results for voltage divider circuit.

fault types	V_1(v)	V_2(v)	V_3(v)	V_4(v)	V_5(v)	S_{15}	S_{35}	S_{45}	diagnose result
$1.3R_2$	8.38	3.65	1.84	1.02	0.68	-5.28	3.27	1.54	×
$1.5R_2$	8.57	3.39	1.72	0.93	0.62	-4.08	2.55	1.45	√
$2R_2$	8.95	2.98	1.53	0.83	0.56	-4.82	2.73	1.52	√
$5R_2$	10.15	1.69	0.86	0.46	0.3	-4.78	2.69	1.49	√
open	11.85	3.5mv	1.8mv	1mv	0.7mv	-5.12	2.75	1.5	√
short	6.03	6.03	2.97	1.66	1.1	-5.63	2.58	1.51	√

5.2. The leapfrog filter circuit

To demonstrate the universality of the proposed method in this paper, another more complicated benchmark circuit (a linear AC circuit) shown in Fig. 3 is considered. It is a leapfrog filter circuit [21] with a cut-off frequency of 1.4 kHz.

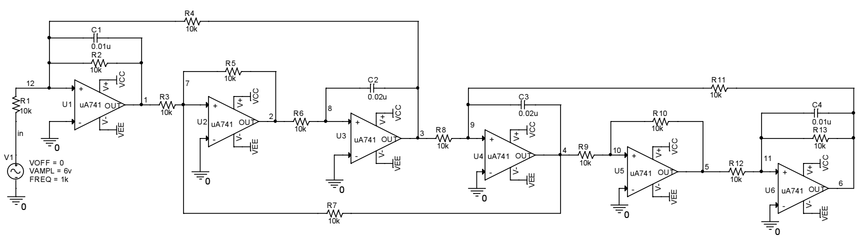

Fig. 3. Leapfrog filter circuit.

Suppose there are 6 test-nodes $\{T_1, T_2, ..., T_6\}$ and 10 possible fault states $\{F_1, F_2, ..., F_{10}\}$ with $(R_3, R_4, R_6, R_8, R_9, R_{10}, R_{11}, R_{12}, C_2, C_3)$ being the faulty components respectively. The parameters of the faulty components increase by 50% of their nominal values. The tolerances of all the non-faulty components are 5%. A sine wave signal with a frequency of 1 kHz, which is in the transition band of the filter circuit, was chosen to be the excitation signal for the CUT.

Both amplitude signal and phase signal are processed by the proposed method. From the MC simulations for the predefined faults under tolerances, $(\theta_i^j, \sigma_i^j, \rho_{ik}^j)$ are computed firstly by statistical methods. Then, the basic test-node is determined to be T_6. After that, the nominal SFC and the SFC ranges of the amplitude signal and phase signal can be calculated and listed in Table 7, Table 8 and Table 9. There are totally $m(m-1)/2 = 45$ fault-pairs. T_1 was selected to be the most informative test-node. There are 39 fault-pairs which can be isolated by S_{16}. Then, T_5 was selected to isolate another 5 fault-pairs. Only (F_3, F_9) cannot be isolated. In fact, the nominal SFC vectors under fault F_3 and F_9 (Table 7) are too close to be distinguished. The final optimum test-nodes set $T^{opt} = \{T_1, T_5, T_6\}$.

Table 7. Nominal SFC for leapfrog filter circuit.

	amplitude signal					phase signal				
	$\overline{S}_{16}$	$\overline{S}_{26}$	$\overline{S}_{36}$	$\overline{S}_{46}$	$\overline{S}_{56}$	$\overline{S}_{16}$	$\overline{S}_{26}$	$\overline{S}_{36}$	$\overline{S}_{46}$	$\overline{S}_{56}$
F_1	0.87	-1.7	-1.29	-1.18	-1.18	24.29	-1.01	-1.04	-1.01	-1.01
F_2	1.05	1.62	1.26	1.16	1.16	22.99	-1	-0.98	-0.99	-0.99
F_3	1.96	1.78	-1.29	-1.19	-1.19	8.15	-0.99	-0.99	-1	-1
F_4	0.99	0.66	0.56	-1.17	-1.17	0.4	-1.01	-1.02	-0.99	-0.99
F_5	-0.47	1.55	1.19	0.97	-1.16	1.4	0.49	0.45	-1	-1
F_6	1.26	-0.88	-0.62	-1.49	1.22	-0.36	-1.23	-1.24	1	1
F_7	-0.61	2	1.53	1.26	1.26	1.39	0.5	0.46	-1	-1
F_8	-0.51	1.51	1.16	0.95	0.95	1.23	0.39	0.44	-1.01	-1.01
F_9	1.97	1.78	-1.23	-1.15	-1.15	8.52	-1.01	-1.02	-1	-1
F_{10}	1.56	2.02	1.56	-1.15	-1.15	0.38	-0.48	-0.5	-1	-1

Table 8. SFC ranges for leapfrog filter circuit (amplitude signal).

	S_{16}	S_{26}	S_{36}	S_{46}	S_{56}
F_1	0.73~1.02	-2~-1.44	-1.51~-1.09	-1.37~-1.01	-1.39~-0.99
F_2	0.24~2.13	0.66~2.96	0.77~1.96	0.79~1.62	0.86~1.53
F_3	1.64~2.35	1.5~2.08	-1.72~-0.96	-1.61~-0.86	-1.58~-0.89
F_4	0.71~1.38	0.28~1.02	0.23~0.84	-1.81~-0.76	-1.79~-0.78
F_5	-0.64~-0.32	1~2.24	0.88~1.59	0.86~1.13	-1.71~-0.79
F_6	0.91~1.71	-1.88~-0.2	-1.14~-0.29	-2.26~-1	1.04~1.46
F_7	-1.41~-0.16	1.3~3.68	1.07~2.66	0.9~1.87	1.01~1.55
F_8	-0.87~-0.18	0.95~2.16	0.73~1.65	0.8~1.12	0.87~1.03
F_9	1.49~2.54	1.31~2.32	-1.79~-0.85	-1.83~-0.69	-1.67~-0.8
F_{10}	1.01~2.56	1.54~2.85	1.24~2.15	-1.91~-0.75	-2.23~-0.57

Fault diagnosis: Suppose $(R_3, R_8, R_9, R_{11}, R_{12}, C_3)$ are the faulty components respectively with the parameters increased by 50% of their nominal values. The parameters of the non-faulty components change around their nominal values independently within the tolerance limits (5%). There are 30 simulations for each fault states. The diagnosis approach proposed in this paper and the FWA in reference [9] are both used to diagnose the faulty CUT. The optimum test-nodes set for FWA is $T^{opt} = \{T_1, T_2, T_6\}$ and the corresponding SFC are $\{S_{16}, S_{26}\}$. The width factor of FWA is 0.1. The fault-pair of (F_3, F_9) still cannot be isolated by FWA. The diagnosis results by the two diagnosis methods are listed in Table 10.

Table 9. SFC ranges for leapfrog filter circuit (phase signal).

	S_{16}	S_{26}	S_{36}	S_{46}	S_{56}
F_1	17.9~37.86	-2.09~-0.49	-2.14~-0.5	-2.12~-0.48	-2.12~-0.48
F_2	16.53~37.38	-2.21~-0.44	-2.16~-0.43	-2.19~-0.46	-2.19~-0.46
F_3	7.21~9.37	-1.27~-0.78	-1.27~-0.78	-1.28~-0.79	-1.28~-0.78
F_4	0.3~0.55	-1.91~-0.55	-1.91~-0.56	-1.97~-0.5	-1.97~-0.5
F_5	0.97~2.05	0.41~0.55	0.36~0.51	-1.59~-0.63	-1.59~-0.63
F_6	-0.59~-0.2	-2.07~-0.81	-2.07~-0.82	0.85~1.17	0.85~1.17
F_7	0.84~2.67	0.22~0.64	0.14~0.61	-2.44~-0.4	-2.44~-0.4
F_8	0.69~2.68	0.07~0.53	0.14~0.57	-2.77~-0.37	-2.77~-0.37
F_9	7.26~10.33	-1.39~-0.73	-1.41~-0.75	-1.43~-0.69	-1.43~-0.69
F_{10}	-6.46~6.31	-0.63~-0.35	-0.65~-0.37	-1.18~-0.84	-1.18~-0.84

Table 10. Diagnosis results for leapfrog filter circuit.

fault states		FWA method		NQD method	
		correct number	correct rate	correct number	correct rate
$1.5R_3$	30	9	30%	23	77%
$1.5R_8$	30	8	27%	25	83%
$1.5R_9$	30	11	37%	26	87%
$1.5R_{11}$	30	6	20%	24	80%
$1.5R_3$	30	11	37%	24	80%
$1.5R_3$	30	12	40%	28	93%

Clearly, the general diagnosis correct rate of FWA is lower than the proposed approach in this paper. The diagnosis coverage of the FWA is less than 40%. Based on the better estimation of the SFC ranges for each fault state, the proposed method can diagnose the faulty CUT more accurately. Furthermore, the correct rate of the proposed method can be further improved by making α (in (17)) bigger, but this maybe lead to a disadvantage of optimum test-nodes selection.

6. Conclusions

While the SFM method can solve the soft-fault diagnosis problem in analog circuits effectively, the challenging tolerance problem remains unsolved. In this paper, the redundancy of the traditional SFC set was proved. By eliminating the redundant information, the computation of SFC was reduced greatly. Based on the proposed approximation condition, the approximating distribution function of NQD was set up. The monotonous and continuous mapping between NQD and standard normal distribution was proved. On the ground of the useful mapping, the estimating formulas about the SFC range were deduced. The NQD approach was combined with SFM to improve the diagnosis coverage of soft-fault under tolerances. Experiments demonstrated that the proposed method is effective and can locate the faulty components accurately.

Though the NQD approach was used to diagnose analog circuits in this paper, it can be used in other applications. For example, in linear regression of two least squares estimates, the slope of the regression line is NQD.

The main disadvantage of the diagnosis methods based on SFM is the high demand of accessible nodes. To construct a SFC, at least two accessible nodes are required. For high diagnosis coverage, more accessible nodes are needed. In the future work, our attention will be focused on another research based on subband decomposition, in which only one node of the CUT is needed to be accessible.

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